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PREFACE

Preface

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Francisco Javier González Acuña, better known as Fico, was born in Tampico, Tamaulipas, México, on January 21st, 1942. He obtained his bachelor's degree in Mathematics from Universidad Nacional Autónoma de México (UNAM) in 1963, and then, at Princeton University, he got a Ph.D. in Mathematics in 1970, under the supervision of the eminent topologist Ralph H. Fox. After that, he came back to México and has been since then a member of Instituto de Matemáticas at UNAM. He has also had a close relationship with Centro de Investigación en Matemáticas, CIMAT, in Guanajuato, México, as a part-time researcher for many years.

Fico has been a leading topologist, and in general, a leading mathematician for over 45 years. He has been a pillar in the development of a research community of mathematics in México, and more particularly, he has played an essential rôle in the development of topology in México. Fico has a deep knowledge of many and varied branches of mathematics, and his contributions to the topology of low-dimensional manifolds are influential and wide ranging. We give here a quick review of some of

Dedicated to Fico González Acuña on his 70th birthday.

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the more important of those contributions, not with the intention of being exhaustive, an impossible task in these few lines, but rather to illustrate the relevance of his work.

His first paper, *Independencia de la métrica en la C^0 topología fina de un espacio de funciones*, was written while he was an undergraduate student at UNAM, and published in 1963 in the now extinct journal *Anales del Instituto de Matemáticas de la UNAM*. There he proved that the space of continuous functions between a topological space X and a metric space Y, with the fine topology, does not depend on the metric of Y, but only in its topology. This is a relevant result which has been overlooked, probably because it was written in Spanish and published in a little-known journal. Recently, this paper has been translated into English and can be consulted at the arXiv, with the title *Independence of the metric in the fine C^0-topology of a function space*.

His second paper, *Dehn's construction on knots*, contains some results from his doctoral thesis. In this groundbreaking paper, he studies homology 3-spheres obtained by Dehn surgery on knots. First, he gives a formula to calculate the μ-invariant of a homology sphere obtained by Dehn surgery on a knot K: this is simply $\mu = \nu \cdot \chi$, where χ is the Arf invariant of K, and ν is an integer indicating the surgery performed; to get this formula, a deep knowledge of topology of 4-manifolds is needed. Then, he announces what was called the *Property P conjecture*, which was also considered independently by R.H. Bing and J.M. Martin. This conjecture says that if K is a non-trivial knot in the 3-sphere, only the trivial Dehn surgery can produce a simply connected manifold. Fico then proves this conjecture for several classes of knots, and for some of these classes, complicated arguments in combinatorial group theory are used. This paper, published in 1970 in the *Boletín de la Sociedad Matemática Mexicana*, was one of the first papers dedicated to Dehn surgery on knots, and influenced a lot of activity in this subject. It is one of the most cited papers in all of the history of the Boletín. The Property P conjecture was finally proved by T.S. Mrowka and P.B. Kronheimer in 2004.

In the 1970s, Fico studied algebraic questions related to the topology of knots and 3-manifolds. In one of his most celebrated articles, *Homomorphs of knot groups*, he gives a positive answer to Problem U in the list of Neuwirth, namely, that a group G is the homomorphic image of the fundamental group of a knot K in S^3, if and only if G is finitely generated and has weight one, that is, it is normally generated by one element. This paper was published in the prestigious *Annals of Mathematics* in 1975. In the same decade, in collaboration with J.M. Montesinos, Fico gave a positive answer to what they call the Fox's Last Problem, proving that there exist n-knot groups, that is, fundamental groups of the complement of knotted spheres S^n in S^{n+2}, for $n \geq 2$, which have infinitely many ends. In the same paper, they characterize the n-knot groups, $n > 2$, which have infinitely many ends. This paper was also published in the *Annals of Mathematics*, in 1978. Both papers contain comments by J. Milnor.

In the 1980s, he came back to the study of Dehn surgery, this time to consider the problem of when Dehn surgery on a knot can produce a reducible manifold, where, in collaboration with H. Short, made a deep study from an algebraic point of view. In the paper *Knot surgery and primeness*, published in 1986 in the *Mathematical Proceedings of the Cambridge Philosophical Society*, they propose the famous Cabling Conjecture, that only surgery on a cable knot along the slope of the cabling annulus can produce a reducible manifold. This conjecture has attracted a lot of attention, and it has been

proved for many classes of knots, but still remains open. Besides results related to the cabling conjecture, this paper contains new and interesting results in combinatorial group theory.

His main research in this decade is the long article *Imbeddings of Three-Manifold Groups*, in collaboration with W. Whitten and published in the *Memoirs of the American Mathematical Society* in 1992. This book studies, in general, the problem of how 3-manifold groups embed in one another and how such embeddings relate to any corresponding essential maps. It contains many interesting results about maps between knot spaces. For example, it is shown that a covering map between knot exteriors is necessarily a cyclic covering. From this, they deduce that the group of a knot can contain a knot group of finite index, only if there is a surgery along this knot which yields a lens-like space, thus connecting this result to questions about Dehn surgery on knots. This book received a long and detailed review, in the *Mathematical Reviews of the AMS*, by K. Johannson, one of the leading figures in 3-manifold topology.

At the end of the 1980s, he became interested in Lusternik–Schnirelmann category, a classical topic in topology, which had been considered earlier by several mathematicians, including R. H. Fox, and is now an active topic of research, considered by many mathematicians working in different areas of topology. The question is, given a topological space X, what is the minimal number of elements that an open cover of X can have, given that each of the open sets is contractible in X? This number is the so called Lusternik–Schnirelmann Category of a space X. Fico and J.C. Gómez-Larrañaga considered this problem for the class of all 3-manifolds. For a closed n-manifold M, it is known that $2 \leq \text{cat}(M) \leq n + 1$. In the paper *Lusternik–Schnirelmann category of 3-manifolds*, published in 1992 in the journal *Topology*, they give a complete description of all 3-manifolds with category 2 or 3, modulo the Poincaré Conjecture. This result is used by L. Bessières, G. Besson, M. Boileau, S. Maillot and J. Porti in the proof of a collapsing theorem, which is the last step in Perelman's proof of Thurston's Geometrization conjecture (*Geometrisation of 3-manifolds*. EMS Tracts in Mathematics, 13. European Mathematical Society (EMS), Zürich, 2010. x+237 pp.).

Over the next two decades, the topic of category and its generalizations has been one of Fico's main research topics, producing over ten papers in collaboration with J.C. Gómez-Larrañaga and W. Heil. For example, in one of the most recent papers, *Amenable category of three-manifolds*, published in the journal *Algebraic and Geometric Topology* in 2013, as the title states, they determine the amenable category of 3-manifolds. A survey of the works of Fico and his coauthors in this topic appears in the present volume.

In the 1990s, Fico continued his interests in the algebraic properties of knot groups, producing interesting results, some in collaboration with W. Whitten in the early 1990s, and others with A. Ramírez in the 1990s and 2000s. One of the main results obtained with Ramírez is a knot-theoretic equivalent of the Kervaire Conjecture, presented in the paper *A knot-theoretic equivalent of the Kervaire conjecture*, and published in the *Journal of Knot Theory and its Ramifications* in 2006. The Kervaire Conjecture says that if G is a non-trivial group, then the group $G * \mathbb{Z}$ cannot be normally generated by one element. They proved that this conjecture is equivalent to what they call the $\mathbb{Z}$-conjecture: Let F be an incompressible, connected, orientable and non-separating surface (an *ICON* surface) properly embedded in the exterior $E(K)$ of a knot K.

Then, the fundamental group of the quotient space $E(K)/F$ is isomorphic to $\mathbb{Z}$. This conjecture remains open. Recently, a student of Fico, J. Rodríguez-Viorato, has proved this conjecture for a class of pretzel knots.

Fico has also purely algebraic interests, and during the last decade has produced in collaboration with A. Díaz-Barriga, F. Marmolejo and L. Román a series of papers on Active Sums, a topic in group theory.

The papers of Fico contain many results, and tend to be produced over long periods of time, only published when they have matured sufficiently. For example, there is a curious fact about the paper *Unsolvable problems about higher-dimensional knots and related groups*, written in collaboration with C. McA. Gordon and J. Simon and published in *L'Enseignement Mathématique* in 2010. In this paper, the authors consider the classes $\mathcal{K}_n$ of fundamental groups of complements of n-spheres in S^{n+2}, for $n \geq 3$, and some other classes of fundamental groups of complements of surfaces in 4-manifolds. They prove the non-existence of an algorithm that can decide if a group in a given class $\mathcal{A}$ actually belongs to a smaller class $\mathcal{B}$, under the assumption that $\mathcal{A} \supset \mathcal{K}_3$. In particular, they prove that there is no algorithm to decide if a given n-knot is trivial, for $n \geq 3$, based on the existence of a group in $\mathcal{K}_n$ with an unsolvable word problem. This last result was also proved by A. Nabutovsky and S. A. Weinberger in 1996. The curious fact is that this research began in the early 1980s but it was not published until 2010.

Fico has results that have not been published yet. For example, back in the 1970s, he defined the concept of an Artin presentation of a group, and showed that any closed orientable 3-manifold has an open book decomposition with a planar page and, using this, he proved that a group is the fundamental group of a closed orientable 3-manifold if and only if it has an Artinian presentation. These results appeared in a set of notes from Iowa university, but have not been published as a paper, though this theory has been developed by several authors, mainly by H. Winkelnkemper.

The influence of Fico amounts to more than his research. He has supervised several doctoral students: María de la Paz Álvarez-Scherer, Víctor Núñez, Enrique Ramírez-Losada, Lorena Armas-Sanabria and Jesús Rodríguez-Viorato. Aside from the theses that he has directly supervised, Fico has influenced all of Mexican mathematicians working in related areas, many of whom have been his coauthors. He has influenced academic life in general, for example, he was editor of the *Boletín de la Sociedad Matemática Mexicana* for many years.

In 2002, when Fico celebrated his 60th birthday, we had the pleasure of organizing *FICOFEST: A Conference in Low Dimensional Topology to celebrate the Sixtieth Birthday of Francisco Javier "Fico" González Acuña*. This was held in the city of Mérida, Yucatán, Mexico, December 6th to 13th, 2002. This meeting began with a series of talks for graduate students, followed by the Congress itself, with 28 invited talks by renowned topologists from around the world. As part of this celebration, a special volume of the *Boletín de la Sociedad Matemática Mexicana* was dedicated to the FICOFEST, which contains 35 papers. This is one of the lengthiest volumes of the Boletín. This volume also contains *A human and intellectual portrait* of Fico, written by José María Montesinos, which surveys aspects of the life and work of Fico not addressed in this preface.

Then in 2012, when Fico celebrated his 70th birthday, we had again the pleasure of organizing a conference in his honor. This was called *School on Knot Theory and 3-manifolds, to celebrate the 70th birthday of Fico González Acuña*, and it was held at CIMAT, in Guanajuato, México, December 17th to 20th, 2012. It was organized by Lorena Armas-Sanabria, Mario Eudave-Muñoz, Fabiola Manjarrez-Gutiérrez, Enrique Ramírez-Losada and Víctor Núñez. The congress had a school format, with mini-courses or series of talks given by Ken Baker, Michel Boileau, Cameron Gordon, Wolfgang Heil, Scott Taylor, Luis Gerardo Valdez-Sánchez and Mariel Vázquez, along with talks by Fabiola Manjarrez-Gutiérrez, Makoto Ozawa, Hamish Short, and some shorter talks given by young mathematicians. This time we embarked again in the production of a volume of the Boletín dedicated to Fico. This volume contains contributions by participants in the conference, along with contributions from mathematicians invited to submit a paper.

The publication of this special volume has involved two invited editors: Mario Eudave-Muñoz of Instituto de Matemáticas at UNAM and of CIMAT, and Víctor Núñez of CIMAT. We heartily thank the authors who responded to our invitation and made this publication possible. All manuscripts received were subjected to rigorous evaluation under the usual rules of the Boletín, and we sincerely thank all the referees for their invaluable assistance.