The mathematical work of Klaus D. Bierstedt

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Gedenkkolloqium, Paderborn, 29/5/2010

Klaus D. Bierstedt, 4.5.1945 – 23.5.2009

Biographical data.

- Born: May 4, 1945.
- Diploma degree in Mathematics, University of Mainz, 1969.
- Dr. rer. nat., University of Mainz, 1971.
- Habilitation, University of Kaiserslautern, 1974.
- Full Professor at the University of Paderborn since 1974 till his death the 23rd of May 2009.

Summary of his mathematical work.

Klaus published 58 articles in international journals and conference proceedings, including **many interesting, motivating, useful surveys.**

Areas of interest

- Topological tensor products and spaces of vector-valued continuous functions.
- Distribution theory.
- Approximation theory.
- Infinite dimensional holomorphy.
- Inductive limits.
- Projective description of weighted inductive limits of spaces of continuous and holomorphic functions.

Organization of Meetings

- Functional Analysis in Paderborn in 1976, 1980, and 1983, with B. Fuchssteiner.
- Functional Analysis meetings in Peñiscola (1990), Essen (1994), and Valencia (2000).
- Meetings on functional analysis in Oberwolfach in the 1980's with H. König and H. Schaefer.
- He initiated joint meetings between the DMV and the American Mathematical Society in Heidelberg in 1993, with the Société Mathématique de Belgique in Liège in 2001 and with the AMS and the ÖMG in Mainz in 2005.

Merits

- Corresponding member of the Societé Royal des Sciences (Liège), elected in 1988.
- Corresponding member of the Real Academia de Ciencias (Madrid), elected in 1999.
- Visiting positions at the Universities of Maryland (USA) 1974, Estadual de Campinas, Sao Paulo (Brasil) 1977, Arakansas, Fayetteville (USA) 1982 and 1986, and Politécnica de Valencia several times since 1991.
- Member of the editorial committee of the journals Mathematische Nachrichten and Arab Journal of Mathematical Sciences.
- Member of the Coordinating Committee of Zbl. MATH and vice-chair of the Scientific Council of FIZ Karlsruhe.
- Supervisor of ten doctoral theses.

Klaus and photos.

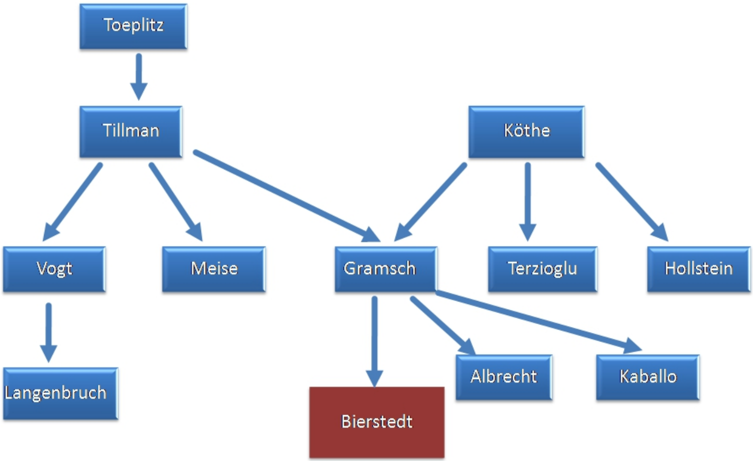
Functional Analysis.

Köthe (about his collaboration with Toeplitz):

"Together we developed the theory of perfect spaces, a counterpart to the theory of Banach spaces. After the Second World War both theories were incorporated into the theory of linear topological spaces, which attained its definitive form in the hands of French mathematicians of the Bourbaki school, after the apparatus of general topology had been sufficiently developed. Since these first papers with Toeplitz, I have remained more or less faithful to this area of Mathematics, also known as functional analysis, which might be characterized as a penetration and further development of classical analysis with the help of topological-algebraical concepts."

J. Dieudonné (LF-spaces), L. Schwartz (Theory of distributions, vector valued distributions, ε-product), A. Grothendieck (DF-spaces, topological tensor products), G. Köthe (sequence spaces).

Mainz, late 1960's and early 1970's.



Mainz, late 1960's and early 1970's.

- Duality of spaces of holomorphic functions, Köthe, 1953.
- Distributions as boundary values of holomorphic functions, Köthe 1952, Tillmann, 1953, 1961,...
- Vogt, 1972-73 (the space $\mathcal{D}'(\mathbb{R}^n)$) and Meise, 1972-73 (the space $S'(\mathbb{R}^n)$).
- Tensor products and vector valued distributions.
- Vogt's conditions (DN) and (Ω).

Mainz, late 1960's and early 1970's.

Mainz, late 1960's and early 1970's.

The early years.

Weighted spaces of vector valued functions and the injective tensor product

- L. Nachbin, Approximation Theory, 1967.
- W.H. Summers, 1969.
- J.B. Prolla, 1971.
- P. Enflo, 1973, Banach spaces without the AP.

Some results

- $CV_0(X, E) = E\varepsilon CV_0(X)$.
- $CV_0(X, E)$ has the approximation property if E and $CV_0(X)$ do.
- $H^\infty(U)$ has the approximation property for the strict topology β.
- Approximation-theoretic localization of Schwartz's approximation property for subspaces of $CV_0(X)$, 1974.
- In 1981 Klaus used results due to Machado and Prolla (1978) to prove his results without completeness assumptions.

The collaboration Bierstedt-Meise.

Background

- **Klaus**: ε-product of Schwartz, weighted spaces of continuous and holomorphic functions.
- **Reinhold**: π-tensor product, nuclear spaces, distribution theory, boundary vales of holomorphic functions.

Topics. Results

- Distributions with values in topological vector spaces, 1973.
- Inheritance of the approximation property by *compactly regular inductive limits,* 1974.
- There are Fréchet Schwartz spaces and DFS-spaces without the AP (1974). Obtained independently by Hogbe-Nlend.

The collaboration Bierstedt-Meise. Infinite holomorphy.

- E metrizable locally convex space, $K \subset E$ compact, $U \subset E$ open.
- $H(K)$ denote the space of holomorphic germs on K.
- $(H(U), \tau_\omega)$ the space of all holomorphic functions on U.

Theorem, 1976-79

The following conditions are equivalent:

- E is Schwartz (resp. nuclear).
- $H(K)$ is a (DFS)-space (resp. a (DFN-space).
- $(H(U), \tau_\omega)$ is a complete Schwartz (resp. s-nuclear) space.

Boland, Colombeau, Dineen, Mujica, Perrot,...

In 1994 he studied Banach valued holomorphic germs on Fréchet Schwartz spaces with Bonet and Peris.

The collaboration Bierstedt-Meise. Weighted inductive limits.

- G is an open subset of $\mathbb{C}^N$.
- $H(G)$ is the space of all holomorphic functions on G.
- A *weight* v on G is a strictly positive continuous function on G.

$$Hv(G) := \{f \in H(G);\ ||f||_v = \sup_{z \in G} v(z)|f(z)| < +\infty\},$$

$$Hv_0(G) := \{f \in H(G);\ vf \text{ vanishes at } \infty \text{ on } G\},$$

endowed with the norm $||.||_v$.

Analogous definitions for $Cv(X)$ and $Cv_0(X)$ for weighted Banach spaces of continuous functions on a locally compact space X.

The collaboration Bierstedt-Meise. Weighted inductive limits.

$\mathcal{V} = (v_n)_n$ is a decreasing sequence of weights on G.

Weighted inductive limits of spaces of holomorphic functions

$$\mathcal{V}H(G) := \mathrm{ind}_n\, Hv_n(G),$$
$$\mathcal{V}_0 H(G) := \mathrm{ind}_n\, H(v_n)_0(G);$$

Fourier Laplace transform of spaces of distributions. Meise's PhD Thesis, 1972. Work of B.A. Taylor, 1971. Complex analysis. Spectral theory...

Projective hulls

$$\overline{V} = \overline{V}(\mathcal{V}) := \{\overline{v} \text{ weight on } G;\ \forall n: \ \sup_G \frac{\overline{v}}{v_n} < +\infty\}.$$

$$H\overline{V}(G) := \{f \in H(G);\ \forall \overline{v} \in \overline{V}: \ p_{\overline{v}}(f) = \sup_G \overline{v}|f| < +\infty\},$$

$$H\overline{V}_0(G) := \{f \in H(G);\ \overline{v}f \text{ vanishes at } \infty \text{ on } G\ \forall \overline{v} \in \overline{V}\},$$

The collaboration Bierstedt-Meise. Weighted inductive limits.

Analogous definitions for spaces of continuous functions on a locally compact space X:

$$\mathcal{V}C(X), \mathcal{V}_0 C(X), C\overline{V}(X), C\overline{V}_0(X),$$

If X is discrete, these are Köthe co-echelon spaces and strong duals of Köthe echelon spaces.

Projective description problem

Under which conditions do the identities

- $\mathcal{V}H(G) = H\overline{V}(G)$
- $\mathcal{V}_0 H(G) = H\overline{V}_0(G)$

hold algebraically and/or topologically?
Same problem for spaces of continuous functions.

The collaboration Bierstedt-Meise. Weighted inductive limits, 1976.

- $\mathcal{V}C(X) = C\overline{V}(X)$ holds algebraically and they have the same bounded sets.
- Consequences of condition (S): $\forall n \, \exists m \;:\; v_m/v_n$ vanishes at ∞. In this case, $\mathcal{V}H(G) = \mathcal{V}_0H(G)$ and it is a (DFS)-space.
- ε product representations of the vector valued spaces.
- Strong regularity conditions on inductive limits. Floret, Neus, Cascales, Orihuela, C. Fernández, Wengenroth.
- If E and F are (DFS)-spaces, then $E\varepsilon F$ is a (DFS)-space.
- **Open Problem.**

Let $E = \mathrm{ind}_n E_n$ be a (DFS)-space. Does

$$E\varepsilon X = \mathrm{ind}_n (E_n \varepsilon X)$$

hold for every Banach space X?

The collaboration Bierstedt-Meise-Summers. TAMS, 1982.

Most influential and most quoted paper of Klaus.

- S. Hansen's approach to the Fundamental Principle of Ehrenpreis, 1981.
- Meise, Taylor, Vogt, solution of a problem of Schwartz, 1990.
- O. v. Grudzinski, convolution operators, 1980.
- Napalkov, weakly sufficient sets for weighted spaces of entire functions. 1982.
- Korobeinik, representing systems.
- Boiti, Nacinovich, the overdetermined Cauchy problem, 1997.
- Taskinen, Jasiczak, continuity of Bergman projection, 2004.

Important discovery:

Use of Baernstein's open mapping lemma to transfer results from spaces of continuous functions to holomorphic functions. Partitions of the unity arguments for continuous functions.

The collaboration Bierstedt-Meise-Summers. Results.

Theorem, 1982

- $\mathcal{V}_0 C(X)$ is a topological subspace of $C\overline{V}_0(X)$.
- $\mathcal{V}_0 C(X) = C\overline{V}_0(X)$ if and only if $\mathcal{V}_0 C(X)$ is complete if and only if the sequence $\mathcal{V}$ is regularly decreasing.
- $\mathcal{V}C(X)$ is boundedly retractive if and only if $\mathcal{V}$ is regularly decreasing.
- The regularly decreasing conditions characterizes when the Köthe echelon spaces are quasinormable.

Theorem, 1982

- If $\mathcal{V}$ satisfies condition (S), then

$$\mathcal{V}_0 H(G) = \mathcal{V} H(G) = H\overline{V}_0(G) = H\overline{V}(G)$$

  algebraically and topologically.

- $\mathcal{V} H(G)$ is complete.
- $\mathcal{V} H(G) = H\overline{V}(G)$ algebraically and they have the same bounded sets.

Open problems in 1983.

Problems, 1983

(1) Is $C\overline{V}(X)$ a (DF)-space?

(2) Is $\mathcal{V}C(X)$ complete?

(3) Characterize when $\mathcal{V}C(X)$ and $C\overline{V}(X)$ coincide topologically.

(4) Characterize distinguished Köthe echelon spaces.

(5) Do the spaces $\mathcal{V}H(G)$ and $H\overline{V}(G)$ coincide topologically?

(6) When do the spaces $\mathcal{V}_0H(G)$ and $H\overline{V}_0(G)$ coincide algebraically or topologically?

Weighted inductive limits of spaces of continuous functions.

Results. Solutions

- $C\overline{V}(X)$ a (DF)-space (Bonet, 1986)
- $\mathcal{V}C(X)$ is complete (Bierstedt, Bonet, 1991)
- Assume that X is locally compact and σ-compact. Then $\mathcal{V}C(X) = C\overline{V}(X)$ holds topologically if and only if $\mathcal{V}$ satisfies condition (D) if and only if the bounded sets of $C\overline{V}(X)$ are metrizable (Bierstedt, Meise, 1986; Bierstedt, Bonet, 1988; Bastin, 1989).
- Characterization of distinguished Köthe echelon spaces $\lambda_1(A)$ in terms of condition (D) (Bierstedt, Bonet, Meise, 1988; Vogt, 1989).

We studied weighted (LF)-spaces of continuous functions in 1994 and weighted (PLB)-spaces of continuous functions with Agethen in 2009.

The density condition and the dual density condition.

Heinrich's density condition **DC** for ultrapowers of locally convex spaces, 1984. Quasinormable spaces and Fréchet Montel spaces have the **DC**.

Susanne Dierolf Habilitation Thesis, 1984.

Theorem, Bierstedt, Bonet, 1988

The following conditions are equivalent for a Fréchet space E.

(a) E has the density condition.

(b) The bounded subsets of E'_b are metrizable.

(c) $\ell_1(E)$ is distinguished.

(d) $\ell_1(E)$ has the density condition.

We studied dual density conditions for (DF)-spaces.
Very good work was contributed by Bastin, J.C. Díaz, Onal and Terzioglu, Galbis and Peris among others.

From a letter of February 1986.

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PADERBORN, DEN February 3, 1986

Dear Pepe:

It's really time that I try to answer at least part of your letters! I am very sorry for the delay, but since I am teaching 8 hours a week, I decided to take most of the time that I would spend during the last weeks to write down §§1-2 of our article in a rather final form and to postpone the letter to the point when this was finished. I am really looking forward to our break (starting February 15) – the two courses in "Analysis" (Advanced Calculus for Freshmen) and "Several Complex Variables" (4 students) took much more time than I had anticipated. During the summer, I will only teach 4 hours per week ("Analysis II"), and then there is a leave from Paderborn coming up next winter! I definitely need it; the last leave was during the summer of '82.

Let's start by answering your letter of January 19. First of all:

You definitely have every reason to be proud of §§1-2!

Of course, I am very happy that the intuition which Reinhold Meise and I had when we introduced condition (D) proved correct – but to show that (D) is necessary as well, which I tried before, would have been impossible without your ideas. I already told you some time ago that, in fact, your part in our paper is much bigger than mine! From time to time, I am happy to contribute "something" of my own, say, a smoother proof or, now, Theorem 9 (which you had suggested).

Some 3 weeks ago, Pietsch was here for a colloquium talk. (He is just spending 5 weeks in West Germany, mainly in Kiel with Hermann König.) I told him about our results and that they are based on Heinrich's article. After the conversation, I feel

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Weighted inductive limits of spaces of vector valued functions. Back to tensor products.

- Taskinen, 1986-88, **problem of topologies of Grothendieck from 1954.**
- **Open Problem, 1976.** Let $E = \mathrm{ind}_n E_n$ be a (DFS)-space. Does $E\varepsilon X = \mathrm{ind}_n(E_n\varepsilon X)$ hold for every Banach space X?
- Floret, Kaballo, Hollstein, 1976-85.
- The complete solution was obtained by Peris in a series of deep results 1993-94. He introduced the spaces **quasinormable by operators**.
- Applications to projective description of weighted inductive limits of vector valued holomorphic functions and spaces of Banach valued holomorphic germs on Fréchet Schwartz spaces. Bierstedt, Bonet, Galbis, Peris, 1993-94.
- Elisabetta Mangino's Thesis, 1995-97.

The Thesis Committee of Elisabetta Mangino.

Weighted inductive limits of spaces of holomorphic functions.

Solution to Problem (5). $\mathcal{V}H(G) \neq H\overline{V}(G)$

- Taskinen's counterexample to Problem (5). Visit to Paderborn in late 1992.
- Bonet, Taskinen, 1995.
- Bonet, Melikhov, 1997.
- Bonet, Vogt, 1999.
- Bonet, Taskinen, 2000 (very artificial examples with $\mathcal{V}$ regularly decreasing). Improved later by Taskinen.

Problem (6) remains open.

Spaces of holomorphic functions. Biduality.

Theorem. Bierstedt, Summers, 1993. Solution to a problem of Duncan

$Hv(G) = Hv_0(G)''$ holds canonically if and only if the closure of the unit ball of $Hv_0(G)$ for the compact open topology absorbs the unit ball of $Hv(G)$.

Theorem. Bierstedt, Bonet, Galbis, 1993.

Let v be a radial weight on a balanced domain G such that $Hv_0(G)$ contains the polynomials. Then

- $Hv_0(G)$ has the metric approximation property and the polynomials are dense.
- $Hv(G) = Hv_0(G)''$ holds canonically and isometrically.

Lusky's work on the isomorphic classification of Banach spaces $Hv(G)$, $G = \mathbb{D}$ or $G = \mathbb{C}$. 1995-2006.
Galbis, Agethen and Wolf published related work.

Weighted (LF)-spaces of continuous or holomorphic functions.

(LF)-spaces

- Vogt, lecture notes on projective imits of (LB)-spaces, 1987, (LF)-spaces, 1990.
- Bierstedt, Bonet, 1994, weighted (LF)-spaces of continuous functions.
- Wengenroth, 1996, every sequentially retractive (LF)-space is acyclic.
- Albanese, Mangino (1996, 2000) Weighted spaces of continuous functions with values in a Fréchet space. Solution of problems Bierstedt, Bonet.
- The projective description does not hold for the Fourier Laplace transform of $A(\mathbb{R})'_b$ (Ehrenpreis, 1970) and of spaces of ultradifferentiable functions of Roumieu type (Bonet, Meise, 2001).
- Bonet, Meise, Melikhov, 2001-2005.

Recent results. The associated weights. Bierstedt, Bonet, Taskinen, Studia Math. 1998.

Associated weights were studied as a tool to characterize properties of weighted inductive limits in terms of the weights. They had been used implicitly by several authors.

$$\tilde{v}(z) := \frac{1}{||\delta_z||} = \frac{1}{\sup\{|f(z)|;\ |f| \leq 1/v \text{on} G\}},\ z \in G.$$

They have been used in

- **Composition operators:** Bonet, Bourdon, Contreras, Galindo, Lindström, Montes, Palmberg, Taskinen, Elke Wolf,...
- **Isometric classification of Banach spaces $Hv(G)$:** Boyd, Rueda, Lusky.
- **Infinite dimensional holomorphy, algebras:** Aron, Carando, Galindo, García, Maestre, Sevilla...

Recent results. The 21st century.

Work with Silke Holtmanns, 1998-2002

- Tensor product representations cannot be applied for spaces of vector valued continuous $CV(X, E)$ or holomorphic functions $HV(X, E)$.
- For every open $G \subset \mathbb{C}^N$, every Nachbin family V of weights on G (generating a topology stronger than the compact-open one) and every quasibarrelled locally convex space E, one has $HV(G; E'_b) = L_b(E, HV(b))$. Related results by Bonet, Friz, 2004.
- They also discussed when an isomorphism $VH(G, E'_b) = L_b(E, VH(G))$ holds for weighted (LB)-spaces $VH(G)$ of holomorphic functions and Fréchet spaces E.

Recent results. The 21st century.

Bierstedt, Bonet, 2003

Based on work by Lusky (1995) and Mattila, Saksman and Taskinen (1997).

We defined a class of radial weights $\mathcal{W}$ on the unit disc such that the projective description holds for (LF)-spaces $\mathcal{V}H(\mathbb{D})$ and $\mathcal{V}_0H(\mathbb{D})$.

The weights in the class $\mathcal{W}$ and our methods were very useful for Elke Wolf's work on Fréchet spaces of holomorphic functions (2004-08).

Recent results. The 21st century.

Bierstedt, Bonet, 2006

Under mild assumptions

- $\mathcal{V}_0 H(G) = \mathcal{V}H(G)$ algebraically implies that $\mathcal{V}_0 H(G)$ is a (DFS)-space.
- $H\overline{V}_0(G) = H\overline{V}(G)$ algebraically implies that $H\overline{V}(G)$ is semi-Montel.
- If the weights are radial and $\mathcal{V}_0 H(\mathbb{D})$ is complete, then it is sequentially retractive.

Bierstedt, Bonet, Taskinen, 2008

Based on work by Lusky (2006).

We show that the projective description holds for weighted inductive limits of spaces of entire functions for radial weights of a special exponential type.

Klaus and Elke Wolf, 2004.

Klaus's Christmas card 2008.

12 / 2008

Dear Pepe, dear Encarna,

best wishes for Christmas 2008

and for a good and successful

New Year 2009

Klaus

It was good to have you here for almost one month

this year. I still plan to spend about a month

(part of April / May) in Valencia next year.

Klaus's favorite mathematical problem.

Problem. Grothendieck, mid 1950's

Is every regular (LB)-space $E = \mathrm{ind}_n E_n$ complete?

What did we/I learnt from Klaus?

- Mathematics is important.
- Abstract functional analysis must be related to analytic problems.
- Teaching and helping young people is essential for Mathematics.
- We must explain what we do, for example writing motivating surveys and stating open problems.

What did we/I learnt from Klaus?

Above all that

FRIENDSHIP IS VERY IMPORTANT.