Early experiences in Number Theory

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I first encountered number theory in the autumn of 1967, during the last academic year of my undergraduate studies at the University of Barcelona. I had enrolled on a course entitled *Teoría de números*, which was to be given by Professor Enrique Linés.

Linés was an analyst who loved to teach undergraduate students, and whom I already knew from his earlier, highly stimulating lectures on Real Analysis. What is more, one of his assistants at the University was Ms. Griselda Pascual, my wonderful mathematics teacher at secondary school.

Linés began his lectures by saying that the purpose of the course was to carry out a joint study of a book that had just arrived in the Faculty Library. The book was, no less, the French translation of Borevich-Shafarevich's *Théorie des Nombres* [3].

As is well known, the late sixties was a tumultuous period everywhere. Unauthorized meetings, riots, fights between students and police, and strikes paralysed academic life at Spanish universities day after day.

Because of this, Linés' expectations for the course would remain unfulfilled, since we could only go through the first chapters of [3]; the local methods presented in chapter 4 and the analytical methods in chapter 5 remained completely untouched.

In 1970, after teaching mathematics for a year at a secondary school, I obtained a scholarship from the Spanish Ministry to take part in an innovative three-year program for training scientific researchers. At the same time, I began to teach as an assistant at the University of Barcelona and at the newly created Autonomous University of Barcelona (in Bellaterra, a small village 22 km from Barcelona).

To begin my research training, I studied the whole of Borevich–Shafarevich's book from scratch, trying to solve as many exercises as possible. This took me the whole of 1971. My next step was spelled out in a passage from Borevich–Shafarevich's book:

> *La théorie du corps de classes décrit la loi de décomposition des diviseurs premiers d'un corps quelconque k de nombres algébriques en facteurs dans une extension K/k si le groupe de Galois de cette extension est abélien* [...] *On connaît trés peu de résultats sur les lois de décomposition des nombres premiers dans les corps dont le groupe de Galois est non abélien.*

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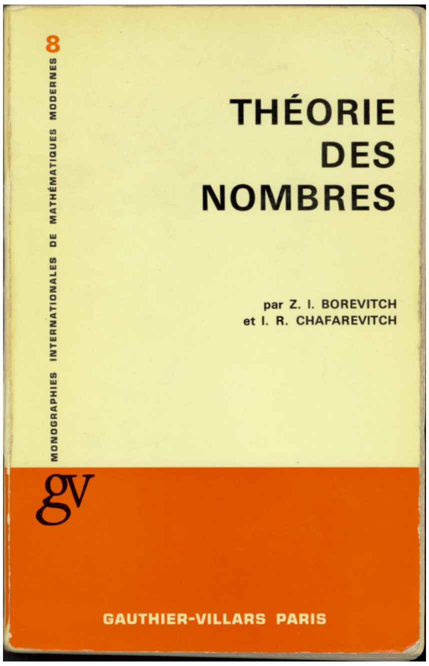

Figure 1: My copy of Borevich–Shafarevich's book *Théorie des Nombres*.

My journey through class field theory would last for the next two years. Luckily I was not alone, because Griselda (who was 20 years my senior) joined me on the project. It took us over a year to study the local theory, through Serre's book *Corps locaux* [6], and almost another year to familiarize ourselves with the global theory, through Artin–Tate's book *Class Field Theory* [1]. Both books were innovative, marvelously written, and presented an extremely profound theory in an accessible way. We studied their pages with great excitement, without even noticing the heat of the summer afternoons. It was not only a matter of understanding the contents of the books, but also of becoming familiar with the variety of tools used in them to obtain the main theorems. (We were blissfully unaware of the fact that the development of class field theory had taken more than 100 years.)

Our encounter with class field experience left Griselda and me exhausted, but we had the impression that we now had a good grounding in the modern developments in the field and might be ready to undertake some sort of personal research. The 100 bibliographical references at the end of Serre's *Corps locaux* were tantalizing, but where should we start?

I took advice from two mathematicians. The first was Jean Dieudonné. In a visit of him to Barcelona, he told me of Jacques Martinet's thesis, which had recently appeared in *Annals de l'Institut Fourier* [4]. Dieudonné was very kind and encouraging. The second was Francesc Tomàs, a Mexican mathematician with Catalan ancestors, who had just enrolled at the Autonomous University of Barcelona. Tomàs recommended that I read Jürgen Neukirch's papers.

Griselda studied Martinet's thesis carefully, in which he proved the existence of normal bases for the ring of integers of dihedral number fields of degree equal to $2p$, p denoting an odd prime. The analogous result in the abelian case had been proven by Hilbert long before.

My choice was Neukirch's paper [5], in *Inventiones Mathematicae*, dealing with the remarkable result that normal algebraic number fields are characterized by their absolute Galois group. (But first, I had to translate the paper from German to Catalan –my mother tongue– since at that time I could not read German fluently.)

The problems studied in those papers were an inspiration for us. Two years later, in April 1975, my former teacher and I were able to defend our theses at the University of Barcelona (becoming, on the same day, the second and third female Doctors in Mathematics at the University).

In May 1977, I moved to Regensburg University in Germany. There, in an ideal environment, I enjoyed the immense privilege of working in number theory with the Research Group conducted by Jürgen Neukirch, Günter Tamme and Manfred Knebusch, for more than three years.

Today, the Number Theory Research Group in Barcelona comprises some 40 people from the University of Barcelona, the Autonomous University of Barcelona, and the Polytechnic University of Catalonia. When I look back on these lines for the *Revista Matemática Iberoamericana*, the question I cannot help asking is this: what would have happened if the French translation of Borevich–Shafarevich's work had not been published in 1967?

References

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