# BIRGIT GRODAL: A FRIEND TO HER FRIENDS

**Andreu Mas-Colell**
Universitat Pompeu Fabra

It was much too soon. We knew it could happen. She participated in the proceedings of the conference in her honor held in Copenhagen in September 2002 with a display of her usual acumen and vigor, but we left worried. I do not know if the time ever comes for anyone, but it had not come for her. It is hard to accept. With the passing of Birgit Grodal the European Economic Association loses its 2005 president, the community of European economists one of its shapers and leading figures, the worldwide community of economic theorists and mathematical economists an outstanding general equilibrium theorist, and her many friends, a friend.

Birgit started her career as a mathematician. She did her Ph.D. under the great Danish convexity specialist William Fenchel. For a whole generation of economists the legendary Fenchel notes on convexity were a source of knowledge and of inspiration. It was most natural to move from Fenchel and convexity analysis to mathematical economics and economic theory, especially under the gentle push of economic and social concerns. This was the road that Grodal took. On it she got connected early to the two cauldrons of mathematical economics research of the late 1960s and early 1970s: Berkeley, where Gerard Debreu, Dan McFadden, Roy Radner, David Gale, Steve Smale, John Harsanyi, William Shephard, were mentoring; and CORE, with Jacques Drèze, Jean François Mertens, Jean Gabszewicz, Werner Hildenbrand, and many others. Both institutions teemed with visitors, including many Americans at CORE and many Europeans at Berkeley. Birgit Grodal became a thriving member of this network, and from these origins a successful research career was launched.

I will attempt to describe and summarize Grodal's contributions to economic theory, all of them falling into the realm of general equilibrium theory broadly understood, under six general headings.

Acknowledgment: I want to acknowledge the help of Karl Vind in the preparation of this text. Note added in proofs: it is with much regret that I learned at the beginning of last July of the passing of Karl Vind, eminent economist and lifelong friend and dear colleague of Birgit Grodal.

## 1. Preference and Demand Theory

Her background in convexity theory and the intellectual environment of the late 1960s made preference and demand theory, on the one hand, and the theory of the core on the other, the points of entry of Grodal into general equilibrium analysis.

In preference and demand theory she carried out a very useful task early on, and she did it at the moment that it was needed. The measure theoretic approach to atomless economies, viewed as a model of perfect competition, which was initiated by Robert Aumann, Karl Vind, and, emphasizing a distributional approach, by Werner Hildenbrand, rested, technically speaking, on the possibility to define topological and metric structures on spaces of agent's characteristics—in particular, on spaces of preferences. In one of her first published papers (modestly called "A Note on...", see Grodal 1974) she carried out the most thorough and general analysis of this matter. Without doubt, it constitutes the definitive contribution.

Samuelson once said that preference and demand theory is for many economists a partial but lifelong devotion. Grodal did not fail to go back to these theories on a number of occasions. For a particularly nice instance we refer to the paper she wrote with Hildenbrand (Grodal and Hildenbrand 1989), where a most simple example of an aggregate excess demand function not satisfying the weak axiom of revealed preference is offered: a type of result with significant implications for the uniqueness and the stability analysis of equilibrium. In the example there are four commodities: two factors of production that do not enter into the utility functions and are initially owned by consumers, and two consumption goods. Suppose that there are at least two consumers that are different (in a precise and natural sense). Then the aggregate excess demand will not satisfy the weak axiom!

## 2. Core Theory

As a tool for the analysis of equilibria, the core is present in Grodal's research throughout her career. But, more specifically, in her first period Grodal made two important contributions to the then emerging core equivalence theorem. In Grodal (1972) she inaugurated, with two companion papers (by David Schmeidler and Karl Vind), an important theme in core analysis. Namely, the idea that if an allocation can be improved upon (that is, it is not in the core), then there are many improving coalitions; and, in fact, improving allocations can be subjected to a number of additional restrictions without losing the validity of the core equivalence theorem. In the quoted contribution Grodal showed that the Theorem would still obtain if the improving coalitions were required to be formed as the union of at most L small clusters of almost identical consumers (here L is the number of commodities).

In the second contribution (Grodal 1975), she extended, in a far-reaching way, a theorem of Debreu on the rate of convergence of the core to the set of Walrasian equilibriua. The result of Debreu was for a type economy (that is, the number of consumers tends to infinity, but their characteristics—preferences and endowments—belong to an a priori given finite set). In contrast, Grodal's result was completely general. I cannot resist recording the key, and beautiful, mathematical result that she established in order to unlock the door to a generalization. (Maybe I am not seeing this in an entirely objective way. At the time we shared an office at Berkeley and I was witness to the weeks of impasse and to the feeling of exhilaration when at last the nut was cracked). It goes as follows: Suppose that for every $N$ you have a set of $2N$ vectors $z^i(N)$ in $R^L$. These vectors add up to zero and, independently of $N$, they are all norm-bounded by a constant $K$. Then for every $N$ you can select a group of $N$ vectors from the collection corresponding to $N$ such that their sum adds up to a vector norm-bounded by $K$ (hence the average individual adjustment to make it zero in the aggregate is of order $K/N$).

## 3. Integral Representations

There was an intensely mathematical but economically grounded and difficult topic on which Birgit worked, on and off, all her professional life, namely integral representations as order-homomorphisms on totally preordered function spaces. The economic motivation of this work comes from the need to determine conditions allowing the representation of suitably defined separable preferences by means of additive utility functions. The challenge that Grodal tackled was the extension of the theory, confined until then to a finite number of components, to an infinite number of components. She had to face, therefore, the need to seek the representation not simply as a sum of component utilities but as an integral of such. Her interest on this topic was aroused at CORE, and, in fact, there is a 1968 CORE working paper with Jean-François Mertens on this matter. Much later the research was gathered into two working papers of the Institute of Economics of the University of Copenhagen that appeared in 1990. It is to be desired that they do not remain unpublished. It is conceivable that with her usual very high self-imposed standards she had ideally planned to elaborate more and to polish further the working papers. Unfortunately, she will not be able to do so, but, in truth, it is not clear that the work needs it. Publication would seem fitting.

## 4. Equilibrium with Coordination

In her mature stage, the heart of the contributions of Grodal were to equilibrium theory, competitive and noncompetitive. For competitive theory we have—beyond the work on the core—the research on clubs and markets that will be

reviewed shortly, and two papers that she wrote with Karl Vind on coordinated equilibrium (see Grodal and Vind 2001). We could qualify the latter concept as a Danish School item (Hans Keiding has also dwelt in it). It constitutes a remarkable extension of the usual competitive equilibrium notion, and it yields a powerful tool for equilibrium analysis.

Another contribution of Grodal to equilibrium theory is contained in her last published paper (see Dierker, Dierker, and Grodal 2002), which is the only one she ever wrote on incomplete markets, a topic that, in fact, she mastered. The paper shows her at her peak. It contains a very interesting example displaying nonexistence of constrained efficient equilibria, and it makes us regret with added intensity, if that was possible, that this will have to be her last word on the matter.

## 5. Clubs and Markets

It is well known that one of the main powers of the Walrasian model of equilibrium is the enormous variety of applications that can conceivably be obtained by a proper interpretation of what constitutes a commodity. The story with time and with uncertainty is familiar, as is the personalized commodities formulation associated with public goods. These interpretations may seem easy ex post, but, in fact, they are far from obvious and in some cases they can only be made after resetting the problem carefully and with much delicacy. This is precisely what Grodal, with Bryan Ellickson, Suzanne Scotchmer, and William Zame, did for "club membership" in one of her latest works (see Ellickson, Grodal, Scotchmer, and Zame 1999), thus generating an equilibrium theory of commodity allocations and club membership that, naturally enough, shares the usual optimality and Core equivalence properties of any Walrasian equilibrium. In a sense, clubs are defined as the characteristics of a commodity, the supply of which is the number of membership slots. To overcome the integer problem, economies with a continuum of agents are considered from the start, although the clubs all have finite membership (thus at equilibrium there is an idealized continuum of clubs). The paper shows how all this can be made to fit together in a demand-supply–like model. It is an impressive piece of work and, to repeat, a substantial increase in the conventional coverage of the Walrasian model.

## 6. General Equilibrium with Imperfectly Competitive Firms

In spite of all her work in Walrasian equilibrium theory, one suspects that this was not the most persistent intellectual love of Birgit Grodal. This attribution will have to go to imperfect competition theory. Taken together the ten or so papers that she published on the latter theory (most of them in collaboration with Egbert Dierker and/or Hildegard Dierker) from 1989 to 2002 constitute the most

sustained research effort of Grodal and the work that displays her mature powers at their best. In fact, her interest in this area starts at the very beginning of her career. If I may be allowed a personal recollection, the session of the European summer meeting of the Econometric Society in 1970 (at Barcelona), where I first met Birgit, was devoted to this very topic and, as discussant, she made the point that later would constitute the motivating kernel for her research in the field.

This point could be called the "problem of the numeraire," and it makes a lot of sense for someone that, like Grodal, would be intellectually inclined to question and probe the foundations of theory. In conventional price-taking theory, shareholders (one or many) are unanimous in desiring that firms profit-maximize. Further, it does not matter in which unit we choose to measure profits. However, in imperfect competition theory, where firms can have market power, it can matter a lot for a profit-maximizing behavior of the firm if we measure profits in one unit or another (say, in terms of its inputs or its outputs). This is only an indication of a deeper problem: The profit maximization objective is itself questionable. For example, shareholders are also consumers and as such their interest on the price level may be at odds with the interest of a manager to charge high prices. As an illustration, think of the case of a single owner, where there is no social choice aggregation problem among owners. Then, obviously, the firm should maximize the surplus of the owner. Except in special situations (e.g., parametric prices, the owner does not supply to the firm or consume its products) this is not equivalent to a profit maximization rule.

Grodal was deeply dissatisfied that imperfect competition theory had glossed over this problem, and on this she was correct. Her (and her coauthors) research approach consisted then in asking which sort of behavior could be postulated on imperfectly competitive firms that did not clash with fundamentals, that is, with the reality of one or several underlying shareholders. In Dierker and Grodal (1999) an ingenious and deep answer is provided. Take the easy case where the owners generate and spend their income in the parametric sector, that is, from and in goods that have fixed prices. Suppose then that the firm chooses a profit maximizing production (with respect to any numeraire in the parametric sector). Trivially this production will have the following property: "The aggregate budget set of shareholders at a different production of the firm does not include in its interior the aggregate consumption of the shareholders at the profit maximizing production." The Dierker and Grodal proposal, under the name of "real wealth maximization," consists of adopting for the objective of the firm, also in the general case (where prices matters), the previous bracketed statement. This objective is immune to the choice of numeraire problem and takes into account the interest of the shareholders. A real wealth maximization production may not exist, but it does under fairly general conditions (guaranteeing that the problem is suitably convex) and it relates well to notions derived from surplus, or compensated surplus, maximization. The concept is rich enough and robust enough to build on it a

theory of imperfect competition, both pure and applied. Grodal and her coauthors have already initiated the way. No doubt this will be an influential line of research in the future.

In a typical European fashion, Grodal had deep academic roots in the institution at which she was affiliated all her life: the University of Copenhagen. All her work stems from there. In turn, she was recognized by Danish and European society. Thus, in the last five years of her life she was a member of the Danish Competition Council, and from 1998, the Academia Europeae. The dedication to both she enjoyed much.

From her Danish base, Grodal has been instrumental in the institutional development of economics in Europe. She had, so to speak, an Econometric Society profile (she was a Fellow of the Society and a member of its Council and Executive Committee for several periods) and was part of the small group of people that promoted with much effort, but also with much satisfaction and much success, the growth of modern quantitative economics at European universities. She was particularly proud of the mathematical economics undergraduate program at the University of Copenhagen, which she helped to launch in 1985. But her mentoring went beyond Denmark. She was Head of the Nordic Doctoral Programme in Economics from 1992 to 1997, and for many years she was involved the European econometric winter meeting, a small conference where promising young economists where selected from all over Europe and given a double opportunity: to present and discuss their work with senior members of the profession and to discover that they were not alone, that there were many other researchers similarly aged and similarly professionally inclined at other European universities. The continuity of the meetings has been preserved to this day.

Birgit Grodal was also very active in the founding and the development of the European Economic Association. She was a member of its council from 1987 to 2000 (except for the years 1990–1991) and was elected vice president in 2002. She would have been its president in 2005. It is our loss that this will not came to pass. But it will always be a great satisfaction to know that she was elected. She was, moreover, instrumental in another momentous event in this institutionalization process of the economics academic profession in Europe: namely, the simultaneous and coordinated celebration of the (summer) European meetings of the European Economic Association and of the Econometric Society. To go ahead with this (in principle, experimentally) was a recommendation of the "Grodal report," the report of a committee that she chaired and which she steered with the consummate ability of a good science manager.

Grodal was determined, even stubborn, and liked an argument. But she was also gracious, kind, and always friendly and full of enthusiasm. I first met Birgit in the August of 1970 European Summer meeting of the Econometric Society, to which I have already referred. I was included in a session chaired by Grodal, who also served as discussant. To the eyes of the second-year graduate student that was I, she was severe with the papers from seasoned members of the profession. But

she was kind to me, the student, in spite of the fact that I was presenting a pretty sophomoric piece of research. It was typical of her attitude towards students.

Birgit, we will miss you.

## References

Dierker, Egbert and Birgit Grodal (1999). "The Price Normalization Problem in Imperfect Competition and the Objective of the Firm." *Economic Theory*, 14, 257–84.

Dierker, Egbert, Hildegard Dierker, and Birgit Grodal (2002). "Nonexistence of Constrained Efficient Equilibria When Markets Are Incomplete." *Econometrica*, 70, 1245–51.

Ellickson, Bryan, Birgit Grodal, Suzanne Scotchmer, and William Zame (1999). "Clubs and the Market." *Econometrica*, 67, 1185–1219.

Grodal, Birgit (1972). "A Second Remark on the Core of an Atomless Economy." *Econometrica*, 40, 581–83.

Grodal, Birgit (1974). "A Note on the Space of Preference Relations." *Journal of Mathematical Economics*, 1, 279–94.

Grodal, Birgit (1975). "The Rate of Convergence of the Core of a Purely Competitive Sequence of Economies." *Journal of Mathematical Economics*, 2, 171–86.

Grodal, Birgit and Werner Hildenbrand (1989). "The Weak Axiom of Revealed Preference in a Productive Economy." *Review of Economic Studies*, 56, 635–39.

Grodal, Birgit and Karl Vind (2001). "Walras Equilibrium with Coordination." In *Economics Essays*, edited by Gerard Debreu, Wilhelm Neuefeind, and Walter Trockel. Springer.